Ruin Analysis in a Discrete-time Sparre Andersen Model with External Financial Activities and Random Dividends

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

In this thesis, we consider a risk model which incorporates multiple threshold levels characterizing an insurer's minimal capital requirement, dividend paying situations, and external financial activities. Our model is based on discrete monetary and time units, and the main quantities of interest are the finite-time ruin probabilities and the expected total discounted dividends paid prior to ruin. We mainly focus on the development of computational methods to attain these quantities of interest. One of the popular methods in the current literature used for studying such problems involves a recursive approach which incorporates appropriate conditioning arguments on the claim times and sizes, and we implement this procedure as well. Furthermore, ruin can occur due to both a claim as well as interest expense accumulation as our model allows the insurer to borrow money from an external fund. In this thesis, we consider only non-stochastic interest rates for both lending and borrowing activities. After constructing appropriate recursive formulae for the finite-time ruin probabilities and the expected total discounted dividends paid prior to ruin, we investigate various numerical examples and make some observations concerning the impact our threshold levels have on finite-time ruin probabilities and expected total discounted dividends paid prior to ruin.

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Table of Contents

1 Introduction and Notation

The classical Cramer-Lundberg model is a foundational mathematical representation of an insurer's surplus process in risk theory. However, despite the tractability and simplicity of the model, it has limitations in terms of applications. Certainly, more complex models are desirable in modern industrial settings.

Bruno de Finetti (1957) first introduced the notion of a dividend strategy and the idea of finding an optimal dividend paying strategy for the insurance risk model. This was followed by other researchers including Karl Borch and Hans Gerber who further explored the problem (see e.g. Borch, 1974 and Gerber, 1979). Recently, Drekic and Mera (2011) published a paper on the ruin analysis of a particular threshold-based dividend payment strategy in a discrete-time Sparre Andersen model (in a discrete-time Sparre Andersen model, claims arrive more generally according to a renewal process instead of a binomial process). Their analysis was an extension of Alfa and Drekic (2007), in which the two researchers considered a delayed Sparre Andersen insurance risk model in discrete time, and analyzed it as a doubly-infinite Markov chain to establish a computational procedure for calculating the joint distribution of the time of ruin, the surplus immediately prior to ruin, and the deficit at ruin.

In this thesis, we generalize the insurance risk model introduced in Drekic and Mera (2011). In actual fact, three additional threshold levels are introduced to depict a minimum surplus level control strategy and external financial activities related to both investment and loan undertakings. Readers are referred to, for example, Li (2009) and Cai and Dickson (2004) for other general investment strategies found in insurance risk models, where the former studied an insurance risk model with risky investments under the assumption that the risky assets follow a Wiener process, and the latter considered a Markov chain based interest rate model. Korn and Wiese (2008) studied optimal investment strategies in an insurance risk model where they also assumed that the risky assets follow a Wiener process. The ultimate objective behind our model is to optimally control the threshold levels so as to minimize the finite-time ruin probability and maximize the expected total discounted dividends paid prior to ruin.

We assume that the number of claims process is a modified discrete-time renewal process with independent, positive, integer-valued interclaim times $\{W_1, W_2, \ldots\}$, where W_1 is the duration from time 0 until the first claim occurs and W_i, $i = 2, 3, \ldots$, is the time between the $(i-1)$-th and i-th claims. Furthermore, we assume $\{W_2, W_3, \ldots\}$ forms an independent and identically distributed (iid) sequence of positive random variables with common probability mass function (pmf) $a_j = Pr\{W_i = j\}$, $j = 1, 2, \ldots, n_a$, and corresponding survival function $A_j = Pr\{W_i > j\} = 1 - \sum_{k=1}^{j} a_k$.

In the *ordinary* Sparre Andersen risk model, a claim is assumed to have occurred at time 0^-, implying that W_1 has the same distribution as the ordinary interclaim times $\{W_2, W_3, \ldots\}$. On the other hand, if W_1 is not a "full" interclaim time, then asymptotically in time the limiting distribution of the forward recurrence time is defined via the pmf $\tilde{a}_j = A_{j-1} / \sum_{k=1}^{n_a} A_{k-1}$, $j = 1, 2, \ldots, n_a$ (see e.g. Karlin and Taylor, 1975, pp. 192-193). This yields another important risk model, namely the *stationary* Sparre Andersen risk model in which W_1 has pmf $\tilde{a}_j$ rather than a_j. As a means of accommodating other possible specifications, we assume that W_1 has a more general pmf denoted by $r_j = Pr\{W_1 = j\}$, $j = 1, 2, \ldots, n_r$. Let $R_j = Pr\{W_1 > j\} = 1 - \sum_{k=1}^{j} r_k$ denote its associated survival function. Through appropriate choice of r_j, it is clear that both the ordinary and stationary Sparre Andersen variants are simply special cases of this more general model, referred to as the *delayed* Sparre Andersen risk model. In this thesis, we focus only on the ordinary Sparre Andersen risk model as generalization to the delayed Sparre Andersen risk model can be carried out without much additional effort.

In what follows, let $\mathbb{Z}$ denote the set of all integers, $\mathbb{Z}^-$ the set of negative integers, $\mathbb{Z}^+$ the set of positive integers, and $\mathbb{N} = \{0\} \cup \mathbb{Z}^+$. For $t \in \mathbb{N}$, we define U_t as the insurer's amount of surplus at time t. With the exception of time 0, U_t represents the amount of surplus at the end of time interval $(t-1, t]$, $t \in \mathbb{Z}^+$, at which point any premiums, deposits, claims, or withdrawals corresponding to this time interval have been received/paid out. Specifically, with respect to the time interval $(t-1, t]$, we adopt the convention that premiums are received at $(t-1)^+$ and any claims and/or withdrawals are applied at t^-. However, unlike premiums, claims, and withdrawals, deposits can be made at both $(t-1)^+$ and t^-. Herein, deposit refers to any cash outflow from the surplus process to the external financial system process (henceforth to be referred to as the external fund), whereas withdrawal refers to any cash inflow from the external fund to the surplus process. In this thesis, the amount of funds present at time t in the external fund of the insurer is denoted by F_t, $t \in \mathbb{N}$, and it is a stochastic process that is fully dependent on the surplus process $\{U_t : t \in \mathbb{N}\}$.

Before proceeding further with the notation and mathematical details, it is essential to clarify what we mean by investment and loan activities through the introduction of four threshold levels, namely ℓ_1, ℓ_2, ℓ_3, and β. If the insurer's surplus level is below ℓ_1, the firm is in need of immediate injection of funds, and these funds come from the external fund process $\{F_t : t \in \mathbb{N}\}$. To differentiate between investment activities and loan activities, we will split the support set of $\{F_t : t \in \mathbb{N}\}$ into two disjoint sets, namely $\Delta_f^1 = \mathbb{N}$ and $\Delta_f^2 = \mathbb{Z}^-$.

When $F_t \in \Delta_f^1$, F_t represents the insurer's investment activities measured in discrete monetary units and the insurer earns interest at a constant rate of κ per period. When $F_t \in \Delta_f^2$, F_t represents the insurer's loan activities and interest expense accumulates at a constant rate of κ' per period. We assume that both κ and κ' are strictly positive. Moreover, we assume that it is the insurer's policy to pay out all the outstanding debt before resuming investment activities, and that the insurer first utilizes its investment assets to make any adjustments to its surplus level before engaging in loan activities.

We assume that $\beta \in \Delta_f^2 \cup \{0\}$, and this represents the lower support value of $\{F_t : t \in \mathbb{N}\}$. With the introduction of β, we redefine Δ_f^2 to be $\emptyset$ when $\beta = 0$ and $\{-1, -2, \ldots, \beta\}$ when $\beta \in \mathbb{Z}^-$. To aid in the understanding of how these processes operate in discrete time, we introduce U_{t^-} and F_{t^-} to represent the surplus and external fund levels immediately after the claim instance but before the withdrawal instance. The threshold ℓ_1 represents the insurer's minimum acceptable surplus level, and if U_{t^-} (corresponding to the time interval $(t-1, t]$) is below ℓ_1 due to a claim, we withdraw or borrow from F_{t^-} to bring U_{t^-} up to level ℓ_1. However, if $F_{t^-} = \beta$, then we can neither withdraw nor borrow more from F_{t^-} even if U_{t^-} is below ℓ_1. Also, if F_{t^-} corresponding to the time interval $(t-1, t]$ drifts below β due to interest expense accumulation, we use U_{t^-} to pay back the difference at t^- as a form of deposit so that F_t is at least kept at its minimum support value of β.

On the other hand, ℓ_2 is a trigger point for investment activities. If $U_t \geq \ell_2$, a constant deposit of size d is paid to the external fund at t^+. Thus, the deposit and withdrawal amounts are also stochastic in the sense that they are dependent on the surplus process. Note that a deposit can be made at both the left and right limits of a time interval. We denote the left limit deposit amount corresponding to the time interval $(t, t+1]$ to be d_t, the right limit deposit amount corresponding to the time interval $(t-1, t]$ to be d_t^c, and the withdrawal amount corresponding to the time interval $(t-1, t]$ to be w_t. Finally, as in Drekic and Mera (2011), if $U_t \geq \ell_3$, a random dividend is paid out to shareholders at t^+. We assume that $\ell_1 \leq \ell_2 \leq \ell_3$.

To sum up, premiums and left limit deposits corresponding to the time interval $(t, t+1]$,

$t \in \mathbb{N}$, are collected and paid out at t^+ according to the following respective (random) rates:

$$p_t = \begin{cases} c & \text{if } U_t < \ell_3, \\ X_t & \text{if } U_t \geq \ell_3, \end{cases}$$

and

$$d_t = \begin{cases} 0 & \text{if } U_t < \ell_2, \\ d & \text{if } U_t \geq \ell_2, \end{cases}$$

where $x_i = Pr\{X_t = i\}$, $i = c_1, c_1 + 1, \ldots, c_2$, denotes the pmf of X_t. In other words, we assume that the dividend rate at time t is not only determined by the surplus level, but also by an additional element of randomness via the distribution of X_t. We refer to $c \in \mathbb{Z}^+$ as the pure (constant) premium and assume that $c_1, c_2 \in \{d, d+1, ..., c\}$ where $d \leq c$, $c_1 \leq c_2$, and $\sum_{i=c_1}^{c_2} x_i = 1$. Clearly, c_1 and c_2 are the respective lower and upper support values of the distribution of the random premium amount at time t, X_t. Correspondingly, we interpret $c - p_t$ as the amount of (randomized) dividends paid to shareholders at time t^+. Note that, by assumption, the probability distribution of X_t is identical for all values of $t \in \mathbb{N}$. Let $\mu = E\{X_0\}$ denote the common mean.

Withdrawals and right limit deposits corresponding to the time interval $(t-1, t]$, $t \in \mathbb{Z}^+$, are made at t^- according to the following respective (random) rates:

$$w_t = \begin{cases} 0 & \text{if } U_{t^-} \geq \ell_1, \\ \min\{\ell_1 - U_{t^-}, \max\{0, F_{t^-} - \beta\}\} & \text{if } U_{t^-} < \ell_1, \end{cases}$$

and

$$d_t^c = \max\{0, \beta - F_{t^-}\}.$$

Figures (1) and (2) depict an example of the simultaneous evolution of both the surplus process and that of the external fund.

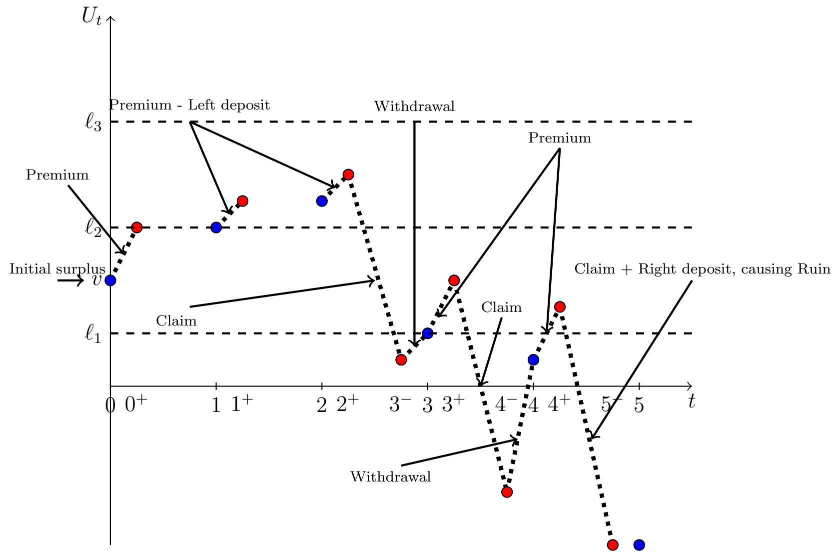

Figure 1: Sample evolution of the surplus process $\{U_t : t \in \mathbb{N}\}$

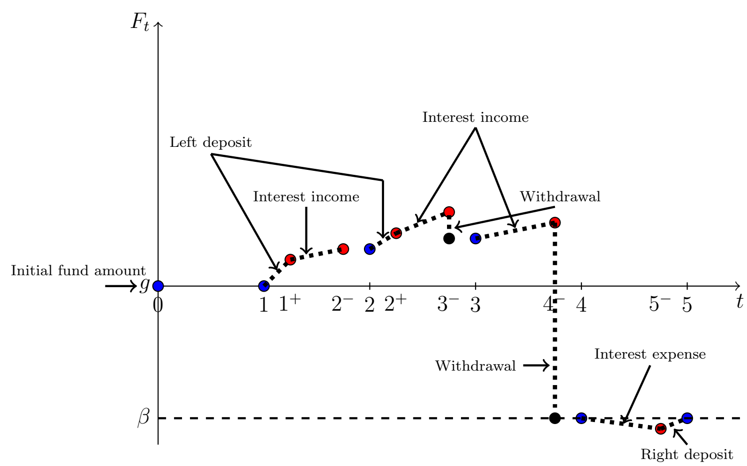

Figure 2: Sample evolution of the external fund process $\{F_t : t \in \mathbb{N}\}$

Beginning at time 0 with an initial surplus level of $v \in \{\ell_1, \ell_1 + 1, \ldots\}$ and an initial external fund amount of $g \in \mathbb{N}$, the insurer's amount of surplus at time t is expressible as

$$U_t = v + \sum_{i=0}^{t-1} p_i - \sum_{i=0}^{t-1} d_i + \sum_{i=1}^{t} w_i - \sum_{i=1}^{t} d_i^c - \sum_{i=1}^{N_t} Y_i,\ t \in \mathbb{N}, \tag{1.1}$$

where N_t is the number of claims occurring by time t and individual claim amounts $\{Y_1, Y_2, \ldots\}$ are assumed to form an iid sequence of positive, integer-valued random variables with common pmf α_j, $j = 1, 2, \ldots, m_\alpha$, and corresponding survival function $\Lambda_j = 1 - \sum_{k=1}^{j} \alpha_k$. We remark that both the interclaim time distribution and the claim amount distribution can be either of finite or infinite support (i.e. $n_a \leq \infty$ and $m_\alpha \leq \infty$).

Following this introduction, Section 2 details the derivation of a recursive formula for the finite-time ruin probability associated with our proposed risk model and demonstrates the simplification of the result to that of Drekic and Mera (2011). Section 3 describes the derivation of a similar recursive formula to compute the expected total discounted dividends paid prior to ruin and likewise demonstrates the simplification of the result to that of Drekic and Mera (2011). Finally, Section 4 discusses some numerical examples and related findings.

2 Finite-Time Ruin Probability

Now that we have introduced the fundamental notation and outlined the definitions for all four thresholds, we begin by examining the finite-time ruin probabilities associated with our risk model. First of all, ruin occurs if and only if $U_t < 0$ for some $t \in \mathbb{Z}^+$ and we denote T to be the time of ruin. In other words, $T = \min\{t \in \mathbb{Z}^+ | U_t < 0\}$ with $T = \infty$ if $U_t \geq 0 \ \forall \ t \in \mathbb{Z}^+$.

In what follows, we are interested in computing the quantity

$$Pr\{T \leq n | U_0 = v, F_0 = g\} = 1 - Pr\{T > n | U_0 = v, F_0 = g\}, \ n \in \mathbb{N}, \tag{2.1}$$

which we refer to as the finite-time ruin probability. To aid in the computation of this quantity, we introduce the following related function:

$$\sigma(u, f, n, m) = Pr\{T > n | U_0 = u, F_0 = f, M_0 = m\}, \ n \in \mathbb{N}, \ u \in \mathbb{Z}, \ f \in \{\beta, \beta + 1, \dots\},$$

where M_t, referred to as the elapsed waiting time counter, denotes the elapsed time at time t since the most recent claim occurrence and its values lie in the set $\{1, 2, \dots, n_a\}$. With the introduction of this function, we remark that (2.1) is calculated via $1 - \sigma(v, g, n, 0)$.

First of all, assuming the occurrence of no claims and no right limit deposits, we need to identify when $U_t \geq \ell_2$ and $U_t \geq \ell_3$ for the first time. We introduce two functions to denote these time points, namely:

$$z_{t,u} = \begin{cases} 0 & \text{if } u \geq \ell_2, \\ \min\{\lfloor \frac{\ell_2 - u - 1}{c} \rfloor + 1, t\} & \text{if } u < \ell_2, \end{cases}$$

and

$$z'_{t,u} = \begin{cases} 0 & \text{if } u \geq \ell_3, \\ \min\{\lfloor \frac{\ell_3 - u - c z_{t,u} - 1}{c - d} \rfloor + z_{t,u} + 1, t\} & \text{if } u < \ell_3, \end{cases}$$

where $\lfloor x \rfloor$, referred to as the *floor function* of x, yields the largest integer less than or equal to x.

To aid us in obtaining a mathematical expression for $\sigma(u, f, n, m)$, we have to examine how the process $\{F_t : t \in \mathbb{N}\}$ evolves over time. Let us first assume that there are no claims or withdrawals to consider. Clearly, F_t is a non-decreasing function of t if $f \in \mathbb{N}$. On the other hand, if $f \notin \mathbb{N}$, F_t could either be a strictly decreasing function of t or perhaps a convex function depending on the values of f, κ' and d. Consequently, if F_t drifts below β, a deposit is forced to be made and this may cause ruin. Interestingly, in this model, ruin can occur due to either a claim or a deposit. This certainly adds more complexity in deriving a formula for $\sigma(u, f, n, m)$, and as a result, we have to introduce a few more functions. One such function is denoted by $o_{t,u,f}$, representing the time point $s \in \{1, 2, ..., t\}$ at which F_s is set to become greater than or equal to 0 for the first time. Obtaining this value is not difficult since, assuming the occurrence of no claims and that $F_s \geq \beta \ \forall \ s \leq o_{t,u,f}$, F_s becomes non-stochastic, the form of which we denote by:

$$\tilde{F}_{s,u,f} = \begin{cases} f(1+\kappa')^s + d^{\kappa'}_{z_{s,u},s} & \text{if } f < 0, \\ f(1+\kappa)^s + d^{\kappa}_{z_{s,u},s} & \text{if } f \geq 0, \end{cases}$$

where $d^x_{k,l}$, $k, l \in \mathbb{N}$, represents the future value of deposits made at times $k^+, (k+1)^+, \ldots, (l-1)^+$ with respect to the interest rate $x > 0$ per period. Clearly, we have $d^x_{k,l} = 0$ for $l \leq k$ and

$$\begin{aligned} d^x_{k,l} &= d(1+x)^{l-k} + d(1+x)^{l-k-1} + ... + d(1+x) \\ &= \frac{d(1+x)[(1+x)^{l-k} - 1]}{x}, \ l > k. \end{aligned}$$

It subsequently follows that

$$o_{t,u,f} = \begin{cases} t & \text{if } \tilde{F}_{i+1,u,f} < 0 \ \forall \ i \in \{0,1,...,t\}, \\ \min\{i \in \{0,1,...,t\} | \tilde{F}_{i+1,u,f} \geq 0\} & \text{otherwise.} \end{cases}$$

Thus, with the introduction of $o_{t,u,f}$, we can express the non-stochastic form of F_t as

$$\hat{F}_{t,u,f} = \lfloor (f(1+\kappa')^{o_{t,u,f}} + d^{\kappa'}_{z_{o_{t,u,f}},u,o_{t,u,f}})(1+\kappa)^{t-o_{t,u,f}} + d^{\kappa}_{\max\{z_{t,u},o_{t,u,f}\},t} \rfloor, \ t \in \mathbb{N}. \tag{2.2}$$

Note that in defining $o_{t,u,f}$, we consider the value of $\tilde{F}_{i+1,u,f}$ instead of $\tilde{F}_{i,u,f}$. This is because for $f < 0$, the function $\tilde{F}_{t,u,f}, t \in \mathbb{N}$, up-crosses level 0 only if a positive amount of deposit is made to the external fund. We stated earlier that left deposits are made at the left limit point of a discrete-time interval. Thus, $\tilde{F}_{i+1,u,f} \geq 0$ for the first time implies that at time i^+, there was a left deposit made to the external fund. In addition, note that (2.2) involves the use of the floor function to calculate the (non-stochastic) value of the external fund at time t. Such an assumption can be viewed as conservative in nature, since any non-integer value of the external fund (which can arise due to interest accumulation) is essentially rounded down.

There is another very important function we next introduce here. It represents the earliest time point when F_t falls below β, again assuming the occurrence of no claims, due to interest expense accumulation. We denote this time point by $c_{t,m,u,f}$ and refer to it as a *calling point*. It is given by

$$c_{t,m,u,f} = \begin{cases} \min\{n_a - m, t\} \ \text{if } \hat{F}_{i,u,f} \geq \beta \ \forall \ i \in \{1,2,...,\min\{n_a - m, t\}\}, \\ \min\{i \in \{1,2,...,\min\{n_a - m, t\}\} | \hat{F}_{i,u,f} < \beta\} \ \text{otherwise.} \end{cases} \tag{2.3}$$

Note that the above function depends on both t and m. As introduced earlier in this section, m represents the elapsed time at time 0 since the most recent claim occurrence.

With these preliminaries in place, we adopt the principle of conditioning on the first claim time as in Cossette et al. (2006) or Drekic and Mera (2011). Measured from time 0, the lower limit of the time until the first claim occurs is 1, but its pmf is now conditional on the value of m. Morever, in evaluating $\sigma(u, f, n, m)$, we will condition on first claim times ranging from 1 up to $c_{n,m,u,f}$, and on the event that the waiting time until the first claim occurs is greater than $c_{n,m,u,f}$. For first claim time instances which take place at or before the calling point, the recursive process used is very similar to that of Drekic and Mera (2011). However, in the event that the waiting time until the first claim occurs is greater than $c_{n,m,u,f}$, the recursive process is performed differently. By doing so, we are essentially denoting $c_{n,m,u,f}$ to be the "new" initial time point, updating the parameters of the function σ, and proceeding with the recursive process. We further explain this situation after we introduce some initial conditions for $\sigma(u, f, n, m)$, namely:

$$\sigma(u, f, n, m) = \begin{cases} 0 & \text{if } u \in \mathbb{Z}^- \text{ or } m = n_a, \\ 1 & \text{if } u \in \mathbb{N},\ n = 0, \text{ and } m = 0, 1, \ldots, n_a - 1. \end{cases}$$

By conditioning on the events outlined above, we get

$$\begin{aligned} \sigma(u, f, n, m) = & \sum_{k=1}^{c_{n,m,u,f}} \frac{a_{k+m}}{A_m} Pr\{T > n | U_0 = u, F_0 = f, M_0 = m, W_1(m) = k\} \\ & + \frac{A_{c_{n,m,u,f}+m}}{A_m} Pr\{T > n | U_0 = u, F_0 = f, M_0 = m, W_1(m) > c_{n,m,u,f}\}, \end{aligned}$$

where $W_1(m)$ is the duration from time 0 until the first claim occurs given that the elapsed waiting time since the most recent claim is m.

At time $k \in \{1, 2, ..., c_{n,m,u,f}\}$, the elapsed waiting time counter is reset to 0 for the next recursion, n is reduced by k, and the"new" initial surplus and external fund amounts are determined by the size of the incurred claim and the premiums received up to time k.

In particular, we obtain

$$\begin{aligned}
&Pr\{T > n | U_0 = u, F_0 = f, M_0 = m, W_1(m) = k\} \\
&= \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}-d(k-z_{k,u})+l+\hat{F}_{k,u,f}-\beta} \alpha_j \sigma(u^\star, f^\star, n-k, 0),
\end{aligned}$$

where

$$\begin{aligned}
u^\star &= u + cz'_{k,u} - d(k - z_{k,u}) + l - j \\
&\quad + \min\{\hat{F}_{k,u,f} - \beta, \max\{0, j - (u + cz'_{k,u} - d(k - z_{k,u}) + l - \ell_1)\}\},
\end{aligned} \tag{2.4}$$

$$f^\star = \max\{\beta, \min\{\hat{F}_{k,u,f}, \hat{F}_{k,u,f} - j + (u + cz'_{k,u} - d(k - z_{k,u}) + l - \ell_1)\}\}, \tag{2.5}$$

and l denotes the value of the sum of the random premiums received up to time k, with corresponding pmf $x_{l,k-z'_{k,u}}$ representing the $(k - z'_{k,u})$-fold convolution of x_l with itself. To evaluate the pmf $x_{a,b}$, $b \in \mathbb{N}$, we define $x_{a,0} = \delta_{a,0}$ (where $\delta_{i,j}$, in general, denotes the Kronecker delta function of κ and j),

$$x_{a,1} = \begin{cases} x_a & \text{if } a = c_1, c_1 + 1, \ldots, c_2 \\ 0 & \text{otherwise,} \end{cases}$$

and for $b = 2, 3, \ldots,$

$$x_{a,b} = \begin{cases} \sum_{j=c_1}^{c_2} x_{j,1} x_{a-j,b-1} & \text{if } a = bc_1, bc_1 + 1, \ldots, bc_2, \\ 0 & \text{otherwise.} \end{cases}$$

The reasoning behind the definitions of the above parameters is that we first consider whether the claim size is substantial enough for the surplus process to fall below its minimum support level ℓ_1. If so, $j - (u + cz'_{k,u} - d(k - z_{k,u}) + l - \ell_1)$ would be a positive quantity.

Then, we consider whether the external fund is able to support the surplus process. We do this by comparing $\hat{F}_{k,u,f} - \beta$ and $j - (u + cz'_{k,u} - d(k - z_{k,u}) + l - \ell_1)$, and choosing the minimum of the two so that we can assure that the external fund does not fall below the maximum level of external funding allowed, β. If $j - (u + cz'_{k,u} - d(k - z_{k,u}) + l - \ell_1)$ is a non-positive quantity, then the surplus process is greater than or equal to ℓ_1 after the claim, in which case, we only need to consider whether $\hat{F}_{k,u,f}$ is below β. If so, $\hat{F}_{k,u,f} - \beta$ is less than 0, and we would subtract $|\hat{F}_{k,u,f} - \beta|$ from the surplus process and add it to the external fund to bring it up to β.

In situations when $W_1(m) > c_{n,m,u,f}$, we will perform a recursion at $c_{n,m,u,f}$ to similarly acquire

$$
\begin{aligned}
&Pr\{T > n | U_0 = u, F_0 = f, M_0 = m, W_1(m) > c_{n,m,u,f}\} \\
&= \sum_{l=(c_{n,m,u,f} - z'_{c_{n,m,u,f},u})c_1}^{(c_{n,m,u,f} - z'_{c_{n,m,u,f},u,f})c_2} x_{l, c_{n,m,u,f} - z'_{c_{n,m,u,f},u,f}} \sigma(u', f', n - c_{n,m,u,f}, c_{n,m,u,f} + m),
\end{aligned}
\tag{2.6}
$$

where

$$
u' = u + cz'_{c_{n,m,u,f},u} - d(c_{n,m,u,f} - z_{c_{n,m,u,f},u}) + l + \min\{0, \hat{F}_{c_{n,m,u,f},u,f} - \beta\} \tag{2.7}
$$

and

$$
f' = \max\{\hat{F}_{c_{n,m,u,f},u,f}, \beta\}. \tag{2.8}
$$

We remark that when $W_1(m) > c_{n,m,u,f}$, there is no claim size to consider at time $c_{n,m,u,f}$. Thus, the only thing that we need to account for is whether $\hat{F}_{c_{n,m,u,f},u,f}$ falls below β. In this case, just enough funds would be withdrawn from the surplus process and added to the external fund to bring it up to β. However, note that $\hat{F}_{c_{n,m,u,f},u,f}$ may not necessarily be below β. If $\hat{F}_{c_{n,m,u,f},u,f} \geq \beta$, then $c_{n,m,u,f} = \min\{n_a - m, n\}$ which implies that either

$n - c_{n,m,u,f} = 0$ or $c_{n,m,u,f} + m = n_a$ in (2.5). This yields an interesting outcome. Given that $W_1(m) \geq c_{n,m,u,f}$ and $\hat{F}_{c_{n,m,u,f},u,f} \geq \beta$, it must be that $u' \geq 0$ at time $c_{n,m,u,f}$. However, if $c_{n,m,u,f} + m = n_a$, then $\sigma(u', f', n - c_{n,m,u,f}, n_a)$ is set equal to 0. On the other hand, if $n < n_a - m$ so that $c_{n,m,u,f} = n$, then $\sigma(u', f', 0, c_{n,m,u,f} + m)$ is set equal to 1. This is a desired result. Putting it altogether, we establish the following recursive formula for $\sigma(u, f, n, m)$:

$$\begin{aligned}
&\sigma(u, f, n, m) \\
&= \sum_{k=1}^{c_{n,m,u,f}} \frac{a_{k+m}}{A_m} \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}-d(k-z_{k,u})+l+\hat{F}_{k,u,f}-\beta} \alpha_j \sigma(u^\star, f^\star, n-k, 0) \\
&+ \frac{A_{c_{n,m,u,f}+m}}{A_m} \sum_{l=(c_{n,m,u,f}-z'_{c_{n,m,u,f},u})c_1}^{(c_{n,m,u,f}-z'_{c_{n,m,u,f},u})c_2} x_{l,c_{n,m,u,f}-z'_{c_{n,m,u,f},u}} \sigma(u', f', n - c_{n,m,u,f}, c_{n,m,u,f} + m).
\end{aligned} \tag{2.9}$$

If we assume that $f = 0, d = 0, \ell_1 = 0, \beta = 0$, and $m = 0$, then the model in consideration is equivalent to the model studied by Drekic and Mera (2011). To verify this, we first note that $c_{n,0,u,0} = \min\{n, n_a\} \; \forall \, n \in \mathbb{Z}^+$. If $n < n_a$, then $\sigma(u', f', 0, n) = 1$ and

$$\frac{A_n}{A_0} Pr(T > n | U_0 = u, F_0 = 0, M_0 = 0, W_1(0) > n) = A_n \sum_{l=(n-z'_{n,u})c_1}^{(n-z'_{n,u})c_2} x_{l,n-z'_{n,u}} = A_n.$$

Conversely, if $n \geq n_a$, then $\sigma(u', f', n - n_a, n_a) = 0$ and

$$\frac{A_{n_a}}{A_0} Pr(T > n | U_0 = u, F_0 = 0, M_0 = 0, W_1(0) > n_a) = A_{n_a} \times 0 = 0.$$

Thus, (2.9) simplifies to become

$$\sigma(u, 0, n, 0) = A_n + \sum_{k=1}^{\min\{n,n_a\}} a_k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}+l} \alpha_j \sigma(u + cz'_{k,u} + l - j, 0, n-k, 0),$$

which is consistent with the result in Drekic and Mera (2011, p. 744).

3 Expected Total Discounted Dividend Payments

The next objective is to derive a similar recursive formula to compute the expected total discounted dividend payments made prior to ruin. The approach we employ essentially borrows from that of Dickson and Waters (2004, Section 5). Let $E\{\mathcal{D}_{v,g}\}$ denote the expected total discounted (i.e. to time 0 according to discount factor $\nu \in (0,1)$ per unit of time) dividends paid prior to ruin, where the random variable $\mathcal{D}_{v,g}$ represents the total discounted dividends paid before ruin starting from an initial surplus of v and an initial level of g in the external fund. In a similar fashion, let us also define the analogous quantity $E\{\mathcal{D}_{v,g,n}\}$ as the expected total discounted dividends paid before ruin occurs or strictly before time $n \in \mathbb{Z}^+$, whichever happens first.

In order to calculate $E\{\mathcal{D}_{v,g}\}$, we construct a computational procedure for calculating $E\{\mathcal{D}_{v,g,n}\}$ and then use the fact that $E\{\mathcal{D}_{v,g,n}\} \to E\{\mathcal{D}_{v,g}\}$ as $n \to \infty$. To aid in the computation of $E\{\mathcal{D}_{v,g,n}\}$, we introduce a function (similar in nature to σ from the previous section) defined by

$$V(u,f,n,m) = E\{\mathcal{D}_{u,f,n}|M_0 = m\},\ n \in \mathbb{Z}^+,\ u \in \mathbb{Z},\ f \in \{\beta, \beta+1, \ldots\},\ m \in \{1,2,\ldots,n_a\}.$$

It clearly follows that $E\{\mathcal{D}_{v,g,n}\}$ can be calculated via $V(v,g,n,0)$.

As with the finite-time ruin probability formula in the previous section, the function $V(u,f,n,m)$ has its own set of initial conditions, namely:

$$V(u,f,n,m) = \begin{cases} 0 & \text{if } u \in \mathbb{Z}^- \text{ or } m = n_a, \\ 0 & \text{if } 0 \le u < \ell_3 \text{ and } n = 1, \\ c - \mu & \text{if } u \ge \ell_3 \text{ and } n = 1. \end{cases}$$

We employ a similar approach as in the previous section by conditioning on values of $W_1(m)$ ranging from 1 up to $c_{n-1,m,u,f}$, and the case where $W_1(m) > c_{n-1,m,u,f}$. By conditioning on these events, we obtain

$$V(u,f,n,m) = \sum_{k=1}^{c_{n-1,m,u,f}} \frac{a_{k+m}}{A_m} E\{\mathcal{D}_{u,f,n,m}|W_1(m)=k\} + \frac{A_{c_{n-1,m,u,f}+m}}{A_m} E\{\mathcal{D}_{u,f,n,m}|W_1(m) > c_{n-1,m,u,f}\}.$$

For $k \in \{1,2,...,c_{n-1,m,u,f}\}$, an expected dividend payment of amount $c-\mu$ would occur at times $z'^{+}_{k,u}, (z'_{k,u}+1)^{+}, ..., (k-1)^{+}$, followed by possible future dividend payments (starting from time k) once the initial claim is applied. Applying the appropriate conditioning arguments, we obtain

$$E\{\mathcal{D}_{u,f,n,m}|W_1(m)=k\} = \sum_{i=z'_{k,u}}^{k-1} \nu^i(c-\mu) + \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}-d(k-z_{k,u})+l+\hat{F}_{k,u,f}-\beta} \alpha_j V(u^\star, f^\star, n-k, 0), \tag{3.1}$$

where $u^\star$ and $f^\star$ are as defined in (2.3) and (2.4), respectively.

For $W_1(m) > c_{n-1,m,u,f}$, we will need to reset the parameters of the function V as we did for the treatment of the finite-time ruin probabilities and base the recursion at $c_{n-1,m,u,f}$. Also, we need to account for the expected dividend payments received at times $z'^{+}_{c_{n-1,m,u,f},u}, (z'_{c_{n-1,m,u,f},u}+1)^{+}, ..., (c_{n-1,m,u,f}-1)^{+}$. Thus, we get

$$\begin{aligned} E\{\mathcal{D}_{u,f,n,m}|W_1(m) > c_{n-1,m,u,f}\} &= \sum_{i=z'_{c_{n-1,m,u,f}}}^{c_{n-1,m,u,f}-1} \nu^i(c-\mu) + \nu^{c_{n-1,m,u,f}} \\ &\times \sum_{l=(c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u})c_1}^{(c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u})c_2} x_{l,c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u}} V(\hat{u}, \hat{f}, n-c_{n-1,m,u,f}, c_{n-1,m,u,f}+m), \end{aligned} \tag{3.2}$$

where

$$\hat{u} = u + cz'_{c_{n-1,m,u,f},u} - d(c_{n-1,m,u,f} - z_{c_{n-1,m,u,f},u}) + l + \min\{0, \hat{F}_{c_{n-1,m,u,f},u,f} - \beta\},$$

and

$$\hat{f} = \max\{\hat{F}_{c_{n-1,m,u,f},u,f}, \beta\}.$$

Note that $\hat{u}$ and $\hat{f}$ are identical in form to (2.6) and (2.7), respectively, with n simply replaced by $n-1$. Combining equations (3.1) and (3.2), we ultimately obtain

$$\begin{aligned} & V(u,f,n,m) \\ = & \sum_{k=1}^{c_{n-1,m,u,f}} \frac{a_{k+m}}{A_m} \Bigg[(c-\mu)\frac{\nu^{z'_{k,u}} - \nu^k}{1-\nu} \\ & + \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}-d(k-z_{k,u})+l+\hat{F}_{k,u}-\beta} \alpha_j V(u^\star, f^\star, n-k, 0) \Bigg] \\ & + \frac{A_{c_{n-1,m,u,f}+m}}{A_m} \Bigg[(c-\mu)\frac{\nu^{z'_{c_{n-1,m,u,f},u}} - \nu^{c_{n-1,m,u,f}}}{1-\nu} + \nu^{c_{n-1,m,u,f}} \\ & \times \sum_{l=(c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u})c_1}^{(c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u})c_2} x_{l,c_{n-1,m,u,f}-z'_{c_{n-1,m,u,f},u}} V(\hat{u}, \hat{f}, n - c_{n-1,m,u,f}, c_{n-1,m,u,f}+m) \Bigg]. \end{aligned} \tag{3.3}$$

As we did in the previous section, let us consider the case when $f=0, d=0, \ell_1=0, \beta=0$, and $m=0$, so that the model in consideration is equivalent to the model discussed in

Drekic and Mera (2011). Under these parameter settings, (3.3) reduces to

$$
\begin{aligned}
V(u,0,n,0) = & \sum_{k=1}^{\min\{n-1,n_a\}} a_k \Bigg[(c-\mu)\frac{\nu^{z'_{k,u}} - \nu^k}{1-\nu} \\
& + \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}+l} \alpha_j V(u+cz'_{k,u}+l-j,0,n-k,0) \Bigg] \\
& + A_{\min\{n-1,n_a\}} \Bigg[(c-\mu)\frac{\nu^{z'_{n-1,u}} - \nu^{n-1}}{1-\nu} \\
& + \nu^{n-1} \sum_{l=(n-1-z'_{n-1,u})c_1}^{(n-1-z'_{n-1,u})c_2} x_{l,n-1-z'_{n-1,u}} V(u+cz'_{n-1,u}+l,0,1,n-1) \Bigg]. \qquad (3.4)
\end{aligned}
$$

We remark that since $c_{n-1,0,u,0} = \min\{n-1,n_a\}$, the square-bracketed term in (3.4) that is pre-multiplied by $A_{\min\{n-1,n_a\}}$ matters only if $c_{n-1,0,u,0} = n-1$ since $A_{n_a} = 0$. Thus, for convenience, we set $c_{n-1,0,u,0} = n-1$ inside this square-bracketed term in (3.4). In addition, (3.4) simplifies further to become:

$$
V(u,0,n,0)
$$
$$
= \begin{cases}
\displaystyle\sum_{k=1}^{\min\{n-1,n_a\}} a_k \Bigg[(c-\mu)\frac{\nu^{z'_{k,u}} - \nu^k}{1-\nu} \\
\displaystyle + \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}+l} \alpha_j V(u+cz'_{k,u}+l-j,0,n-k,0) \Bigg] & \text{if } z'_{n-1,u} < z'_{n,u}, \\[2ex]
\displaystyle\sum_{k=1}^{\min\{n-1,n_a\}} a_k \Bigg[(c-\mu)\frac{\nu^{z'_{k,u}} - \nu^k}{1-\nu} \\
\displaystyle + \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}+l} \alpha_j V(u+cz'_{k,u}+l-j,0,n-k,0) \Bigg] \\
\displaystyle + A_{\min\{n-1,n_a\}} \sum_{i=z'_{n-1,u}}^{n-1} \nu^i (c-\mu) & \text{if } z'_{n-1,u} = z'_{n,u}.
\end{cases}
$$

(3.5)

The logic behind (3.5) is as follows. If $z'_{n-1,u} < z'_{n,u}$, then $z'_{n,u} = n$. This implies that there are no dividend payments before time n, and thus, (3.2) becomes 0. On the other hand, if $z'_{n-1,u} = z'_{n,u}$, there is a guaranteed dividend payment at time $n-1$ and

$$\sum_{l=(n-1-z'_{n-1,u})c_1}^{(n-1-z'_{n-1,u})c_2} x_{l,n-1-z'_{n-1,u}} V(u + cz'_{n-1,u} + l, 0, 1, n-1) = c - \mu.$$

Thus, in the second equation of (3.5), the quantity

$$A_{\min\{n-1,n_a\}} \sum_{i=z'_{n-1,u}}^{n-1} \nu^i (c-\mu)$$

can be rewritten as

$$A_{n-1} \sum_{i=z'_{n,u}}^{n-1} \nu^i (c-\mu).$$

Also, consider the expression

$$\sum_{k=n}^{n_a} a_k \sum_{i=z'_{k,u}}^{n-1} \nu^i (c-\mu). \tag{3.6}$$

If $z'_{n,u} = z'_{n+1,u} = \ldots = z'_{n_a,u}$, then certainly we can replace $z'_{n+1,u}, z'_{n+2,u}, \ldots, z'_{n_a,u}$ with $z'_{n,u}$ in (3.6). Otherwise, $z'_{n,u} = n$, and since $\min\{z'_{n,u}, z'_{n+1,u}, \ldots, z'_{n_a,u}\} = z'_{n,u}$, (3.6) evaluates to 0. In either case, (3.6) becomes

$$\sum_{k=n}^{n_a} a_k \sum_{i=z'_{k,u}}^{n-1} \nu^i (c-\mu) = \sum_{k=n}^{n_a} a_k \sum_{i=z'_{n,u}}^{n-1} \nu^i (c-\mu) = A_{n-1} \sum_{i=z'_{n,u}}^{n-1} \nu^i (c-\mu).$$

Substituting these results into (3.5) and simplifying ultimately yields

$$\begin{aligned} V(u,0,n,0) &= (c-\mu) \sum_{k=1}^{n_a} a_k \frac{\nu^{\min\{z'_{k,u},n\}} - \nu^{\min\{k,n\}}}{1-\nu} \\ &+ \sum_{k=1}^{\min\{n-1,n_a\}} a_k \nu^k \sum_{l=(k-z'_{k,u})c_1}^{(k-z'_{k,u})c_2} x_{l,k-z'_{k,u}} \sum_{j=1}^{u+cz'_{k,u}+l} \alpha_j V(u + cz'_{k,u} + l - j, 0, n-k, 0). \end{aligned}$$

Again, the result coincides with that of Drekic and Mera (2011, p. 745).

4 Numerical Examples and Related Findings

In this section, we further explore the proposed risk model through some numerical examples. Examples we consider here are based on the ordinary Sparre Andersen risk model. First, we introduce a set of interclaim time distributions to study, namely:

$$\text{(a) } a_j = \begin{cases} (2/11)(9/11)^{j-1} & \text{if } j = 1, 2, ..., 24, \\ (9/11)^{24} & \text{if } j = 25, \end{cases} \tag{4.1}$$

$$\text{(b) } a_j = 1/10, \;\; j = 1, 2, ..., 10, \tag{4.2}$$

$$\text{(c) } a_j = \frac{1}{1-(39/50)^{25}} \binom{25}{j} (22/50)^j (39/50)^{25-j}, \;\; j = 1, 2, ..., 25, \tag{4.3}$$

$$\text{(d) } a_j = \begin{cases} (0.645)(1/2)^j + (0.355)(1/12)(11/12)^{j-1} & \text{if } j = 1, 2, ..., 14, \\ (0.645)(1/2)^14 + (0.355)(1/12)(11/12)^{14} & \text{if } j = 15, \\ (0.355)(1/12)(11/12)^{j-1} & \text{if } j = 16, 17, ..., 49, \\ (0.355)(11/12)^{49} & \text{if } j = 50. \end{cases} \tag{4.4}$$

We note that (a) is the pmf of a truncated geometric distribution with $n_a = 25$, (b) is the pmf of a uniform distribution on $\{1, 2, \ldots, 10\}$, (c) is the pmf of a zero-truncated binomial distribution with $n_a = 25$, and (d) is the pmf of a mixture of two truncated geometric distributions with $n_a = 50$. We note that the means are essentially equal to 5.5 for all four interclaim time distributions, but their variability differs with (c) being the least variable and (d) being the most variable.

As for the random premium distribution in effect, we consider a degenerate distribution with all the probability mass on 2 (i.e. $c_1 = c_2 = 2$, so that $x_2 = 1$). In terms of the claim size distribution, we consider a discretized version of the Pareto distribution with mean 10 given by

$$\alpha_j = \left(1 + \frac{j-1}{30}\right)^{-4} - \left(1 + \frac{j}{30}\right)^{-4}, j \in \mathbb{Z}^+. \tag{4.5}$$

The following observations are made concerning the results in Tables 1 to 6, in which interclaim time distribution (a) was used throughout:

(1) In Tables 1 and 4, we assumed that $c = 5, v = 10, g = 0, d = 1, \nu = 0.75, \kappa = 0.01, \kappa' = 0.02, \ell_1 = 0, \ell_2 = 20$, and $\ell_3 = 50$. Under these circumstances, changing the maximal level of external funding allowed resulted in a monotone behaviour in our two performance measures. As we increased $|\beta|$, the finite-time ruin probabilities decreased monotonically for all $n \leq 100$, whereas the expected total discounted dividends paid before ruin increased monotonically. It seems that the benefit of having more funds available outweighs the borrowing costs under the specific setting we considered here. We point out that in Table 4 (as well as Tables 5, 6, 8, and 10) the minimum time point n required to achieve convergence (to six significant digits) of $V(10, 0, n, 0)$ to $E\{\mathcal{D}_{10,0}\}$ is italicized and appears in parentheses next to its corresponding value.

(2) In Tables 2 and 5, we assumed that $c = 5, v = 10, g = 0, d = 1, \nu = 0.75, \kappa = 0.01, \kappa' = 0.02, \beta = -10, \ell_2 = 20$, and $\ell_3 = 50$. Changing the minimal capital requirement level ℓ_1 resulted in a negative effect on the finite-time ruin probabilities. As we increased ℓ_1, the finite-time ruin probabilities monotonically increased for all $n \leq 100$. On the other hand, however, the expected total discounted dividends paid before ruin increased as we increased ℓ_1. Artificially requiring the level of the surplus process to be at a certain positive level prompts more borrowing and this generates higher interest expense. Thus, ruin is more likely to occur. The expected total discounted dividends paid prior to ruin increased since the surplus process is now more likely to reach the dividend payment trigger level ℓ_3, as the surplus level is kept at a higher level more of the time.

(3) In Tables 3 and 6, we assumed that $c = 5, v = 10, g = 0, d = 1, \nu = 0.75, \kappa = 0.01, \kappa' =$

$0.02, \ell_1 = 0, \beta = -10$, and $\ell_3 = 50$. Increasing the investment trigger level ℓ_2 resulted in increasing finite-time ruin probabilities and increasing the expected total discounted dividend payments prior to ruin. As we increase ℓ_2, we are delaying investments and this leads to a negative effect on the finite-time ruin probabilities since the external fund earns interest while the surplus process does not. Nevertheless, the surplus process is kept at a higher level as we increase ℓ_2, and thus, the surplus process is more likely to reach ℓ_3, resulting in higher expected total discounted dividend payments prior to ruin.

The following observations are made concerning the results in Tables 7 and 8, in which interclaim time distributions (a) to (d) were each studied:

(4) We assumed that $c = 5, v = 10, g = 0, d = 1, \nu = 0.75, \kappa = 0.01, \kappa' = 0.02, \ell_1 = 0, \ell_2 = 20, \ell_3 = 50$, and $\beta = -10$. In an effort to investigate the effects of variability in the choice of interclaim time distribution, we observed that the finite-time ruin probabilities were highest for interclaim time distribution (d) and lowest for interclaim time distribution (c) for all $n \leq 100$. The expected total discounted dividends paid prior to ruin ended up being highest for (a) and lowest for (d).

The following observations are made concerning the results in Tables 9 and 10, in which interclaim time distribution (b) was used throughout:

(5) In (1), we observed that the ability to borrow more money from the external fund had positive effects on both the finite-time ruin probabilities and expected total discounted dividends paid prior to ruin. This begs the question as to whether an insurer can continue to borrow more and more money and still produce a positive impact on the business. To investigate this matter further, we increased κ' to 0.30 and varied β from -10 to -30 in increments of size 10. Under this revised setting, not only has the monotone behaviour of the finite-time ruin probabilities changed, but also the effects of increasing $|\beta|$ have changed. From Table 9, note that when $n = 10$, $\beta = -10$ yields the highest

ruin probability. However, for $n = 25$ and onwards, $\beta = -10$ produces the lowest ruin probabilities and $\beta = -30$ has the highest ruin probabilities. On the other hand, the expected total discounted dividends paid before ruin were still highest for $\beta = -30$ and lowest for $\beta = -10$, although the difference was rather minimal.

Finite-time ruin probabilities

	n=25	n=50	n=75	n=100
$\beta = 0$	0.174830	0.196614	0.204672	0.207823
$\beta = -10$	0.109811	0.128569	0.136029	0.139131
$\beta = -20$	0.072636	0.0886259	0.0954679	0.0984981

Table 1: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, interclaim time distribution (a)

Finite-time ruin probabilities

	n=25	n=50	n=75	n=100
$\ell_1 = 0$	0.109811	0.128569	0.136029	0.139131
$\ell_1 = 10$	0.112201	0.132375	0.140743	0.144394
$\ell_1 = 20$	0.115284	0.137842	0.147714	0.152290

Table 2: $v = 10$, $g = 0$, $c = 5$, $\ell_2 = 20$, $\ell_3 = 50$, $\beta = -10$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, interclaim time distribution (a)

Finite-time ruin probabilities

	n=25	n=50	n=75	n=100
$\ell_2 = 10$	0.108725	0.125772	0.132366	0.135042
$\ell_2 = 20$	0.109811	0.128569	0.136029	0.139131
$\ell_2 = 30$	0.111336	0.132243	0.140858	0.144556

Table 3: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_3 = 50$, $\beta = -10$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, interclaim time distribution (a)

Expected total discounted dividends paid prior to ruin	
$\beta = 0$	0.248444 (*60*)
$\beta = -10$	0.252225 (*59*)
$\beta = -20$	0.254015 (*55*)

Table 4: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, $\nu = 0.75$, interclaim time distribution (a)

Expected total discounted dividends paid prior to ruin	
$\ell_1 = 0$	0.252225 (*59*)
$\ell_1 = 10$	0.259287 (*57*)
$\ell_1 = 20$	0.294781 (*56*)

Table 5: $v = 10$, $g = 0$, $c = 5$, $\ell_2 = 20$, $\ell_3 = 50$, $\beta = -10$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, $\nu = 0.75$, interclaim time distribution (a)

Expected total discounted dividends paid prior to ruin	
$\ell_2 = 10$	0.210858 (*57*)
$\ell_2 = 20$	0.252225 (*59*)
$\ell_2 = 30$	0.331537 (*56*)

Table 6: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_3 = 50$, $\beta = -10$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, $\nu = 0.75$, interclaim time distribution (a)

Finite-time ruin probabilities

	n=25	n=50	n=75	n=100
(a)	0.109811	0.128569	0.136029	0.139131
(b)	0.0739737	0.0893741	0.0955145	0.0980538
(c)	0.0577812	0.0719240	0.0775949	0.0799444
(d)	0.198521	0.225533	0.236586	0.241374

Table 7: $v = 10$, $g = 0$, $c = 5$, $\beta = -10$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$

Expected total discounted dividends paid prior to ruin

(a)	0.252225 (*59*)
(b)	0.249026 (*69*)
(c)	0.247518 (*56*)
(d)	0.227710 (*58*)

Table 8: $v = 10$, $g = 0$, $c = 5$, $\beta = -10$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.02$, $d = 1$, $\nu = 0.75$,

Finite-time ruin probabilities

	n=10	n=25	n=50	n=75	n=100
$\beta = -10$	0.0543815	0.0844038	0.107521	0.118964	0.125726
$\beta = -20$	0.0456417	0.0983912	0.131307	0.140746	0.144196
$\beta = -30$	0.0440729	0.103629	0.132196	0.140967	0.144267

Table 9: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.30$, $d = 1$, interclaim time distribution (b)

Expected total discounted dividends paid prior to ruin

$\beta = -10$	0.247093 (*56*)
$\beta = -20$	0.247106 (*56*)
$\beta = -30$	0.247143 (*51*)

Table 10: $v = 10$, $g = 0$, $c = 5$, $\ell_1 = 0$, $\ell_2 = 20$, $\ell_3 = 50$, $\kappa = 0.01$, $\kappa' = 0.30$, $d = 1$, $\nu = 0.75$, interclaim time distribution (b)

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